Educ 506
Fall 2012
Linear Relations Unit Plan for Math 10C
Friday, October 12, 2012
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1. Introduction

   The following unit plan is a blueprint for teaching the parts of Math 10C pertaining to lines and linear relations, preparing students for Linear Algebra and Calculus and various applications. The modern understanding of lines as represented by equations is almost certainly due to Rene Descartes (1596-1650) who invented the Cartesian plane but lines in general were also a favourite topic of the Ancient Greeks approximately two millenia earlier.

   With the birth of the scientific method in about the 17th century an algebraic understanding of lines allowed for many discoveries such as Boyle's law (the volume of a fixed quantity of gas is proportional to the inverse of the pressure, with temperature held constant) and Charles' Law (the volume of a gas is proportional to the temperature, with pressure held constant.)

2. Goals, outcomes, intentions

   (a) To be able to pass between the various forms of the equation of a line and a graphical representation of that line.

   (b) To be able to calculate the slope of a line

   (c) To be able to infer qualitative information about a line from its slope

   (d) To understand what is a linear relation

   (e) To be able to differentiate between linear and non-linear relations

   (f) To develop an appreciation for the beauty of mathematics

   (g) To develop skill in each of the seven categories listed below: PS, R, V, CN, C, T, EM

   (h) To develop a positive attitude and an affective bridge concerning mathematics.

3. Misconceptions - Research on this?

   (a) Any line has a slope - student fails to consider vertical lines

   (b) To any line is a corresponding linear relation - student fails to consider vertical lines

   (c) Any two lines intersect in a single point - student fails to consider parallel or coincident lines

4. Assessment what and how:

(a) of learning - summative

- Pre- and post-tests
- in-class work and homework from textbook McGraw-Hill-Ryerson Mathematics 10 http://www.mhrmath10.ca/ [2]

(b) as learning - formative

- problem-solving
- providing feedback on students' work

(c) through learning - formative

- class participation - reward students for courage (eg. trips to the blackboard) more than for aptitude. Keep a tally of trips to the blackboard.
- questions communicated from student to teacher

5. Motivation Why do students need to know this?

(a) Proportional reasoning as an essential, everyday skill

(b) Engineering

(c) Math 20, Math 30, Math 31 and university math

(d) Rational thinking skills

(e) Practical skills: converting between units, calculating tax, tip; understanding correlations, cause-and-effect relationships

(f) Learn to be open-minded to new perspectives

(g) Appreciate the beauty of mathematics

6. Context within the Curriculum - Alberta Mathematics Program of Studies (POS) [3]

(a) Prerequisites Use formative assessment to establish competence

(b) Solve Problems Involving Linear Measurement (POS Math 10C)

(c) Cartesian plane: pass between ordered pairs and points of the plane (The Cartesian Plane is first introduced in Grade 6 according to the POS)

(d) Solving linear equations (Applying preservation of equality to solve equations is studied in Grade 7, according to the POS.)

(e) Need this for:

(f) Linear algebra (see eg. POS Math 10C Relations and Functions Topic 9)

(g) Trigonometry (see POS Math 10C)

(h) Calculus

7. Codes:

C	Communication
PS	Problem Solving
CN	Connections
R	Reasoning
ME	Mental Mathematics and Estimation
T	Technology
V	Visualization

I. **Lesson: Conversions Between Units**

(1) **Position in POS**:
Measurement Topic 2 [C, ME, PS]

(2) **General Outcome**:
Develop algebraic and graphical reasoning through the study of lines and linear relations.

(3) **Specific Outcomes**:
Apply proportional reasoning to problems that involve conversions between SI and imperial units of measure.

(4) **Learning Objective**:

i. Explain how proportional reasoning can be used to convert a measurement within or between SI and imperial systems.

ii. Solve a problem that involves the conversion of units within or between SI and imperial systems.

iii. Verify, using unit analysis, a conversion within or between SI and imperial systems, and explain the conversion.

iv. Justify, using mental mathematics, the reasonableness of a solution to a conversion problem.

v. Explain how a non-vertical line through the origin corresponds to an equation $y = mx$. Explain that m is called the *constant of proportionality* or *slope*.

(5) **Pre-Assessment**:
Check that students know how to take linear measurements.

(6) **Activity**:
Analyze graphs such as Figure I6 at the end of this document:

(7) **Materials**:
Graph paper, ruler, graphing calculator

(8) **Assessment**:
Have students measure:

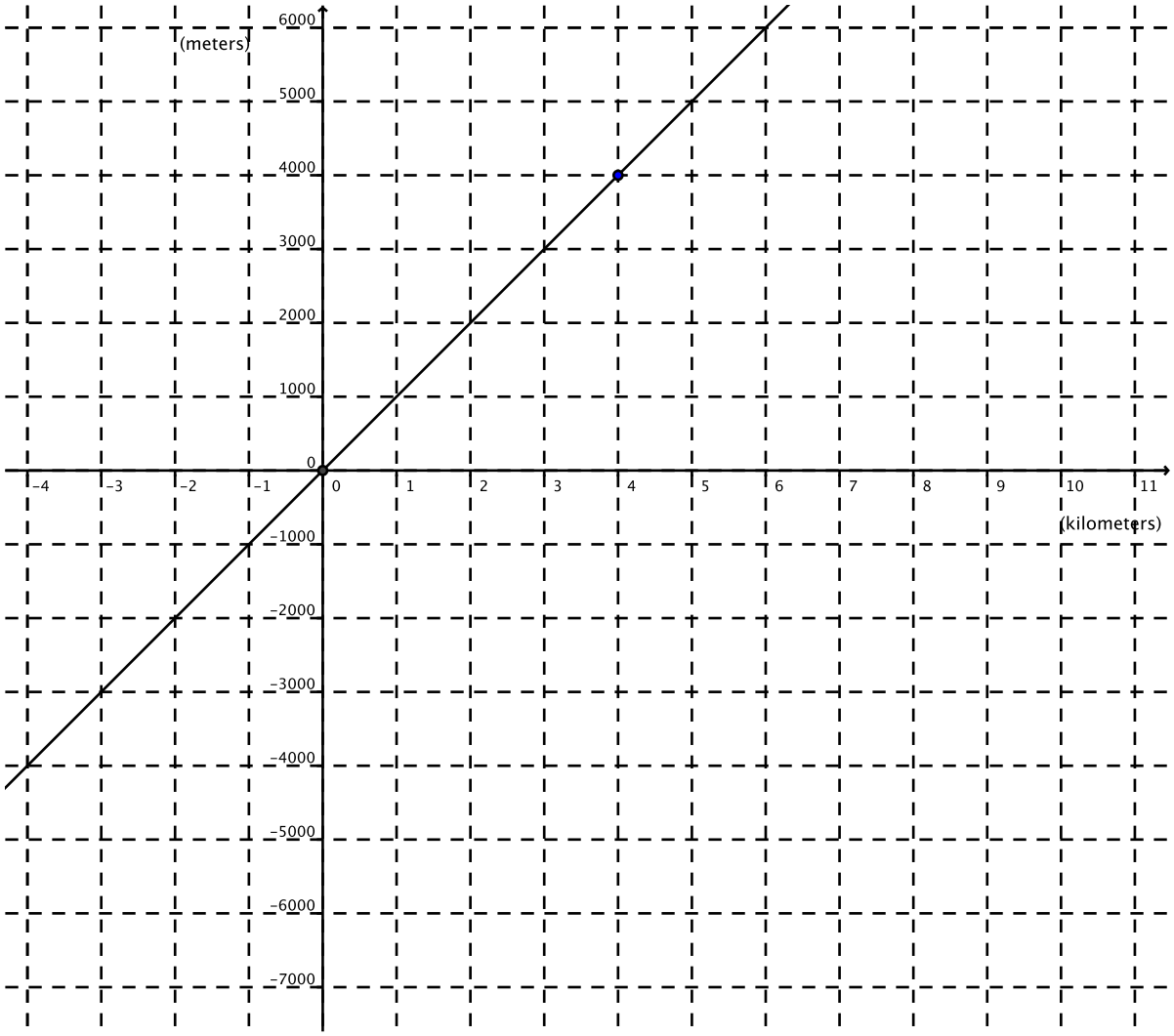
i. the length of their pencils, in centimeters

ii. the width of their thumb, in millimeters

iii. the spacing between lines of a sheet of ruled paper, in millimeters

Have students convert these measurements 1) all into centimeters (cm); 2) all into millimeters (mm); and 3) all into inches, given that 1 cm = 10 mm and 1 inch = 2.5 centimeters.

Assign problems from textbook Unit One "Measurement" - Relating Measurement Units, p. 7.


(meters)
6000
5000
4000
3000
2000
1000
0
−1000
−2000
−3000
−4000
−5000
−6000
−7000
−4
−3
−2
−1
0
1
2
3
4
5
6
7
8
9
10
11
(kilometers)

II. **Lesson: Lines passing through the origin**

(1) **Position in POS**:
Relations and Functions Topic 3 [PS, R, V]

(2) **General Outcome**:
Develop algebraic and graphical reasoning through the study of lines and linear relations.

(3) **Specific Outcome**:
Demonstrate an understanding of slope with respect to line segments and lines.

(4) **Learning Objective**:

i. Classify lines in a given set as having positive or negative slopes.
ii. Explain the meaning of the slope of a horizontal or vertical line.
iii. Explain how the slope of a line through the origin is the tangent of the angle it makes with the positive x-axis.

(5) **Pre-Assessment**:
Give a few problems from the textbook which test understanding of lesson One.
Give an exercise from the textbook which allows students to demonstrate their understanding of the correspondence between ordered pairs and points of the Cartesian plane. There are online resources which test and develop this skill in the context of a game; invite students to visit the site http://games.flyingcoloursmaths.co.uk/games/battle prior to the lesson.

(6) **Activity**: Calculate the slopes of the four lines in Figure II6 at the end of this document:
Have the students differentiate these lines in terms of steepness, like steepness of a hill. Explain how for each of these lines $m = y/x$ is the same for any (x, y) on the line, $x \neq 0$.
Explain that the x-axis is the line $y = 0 = 0x$ so $m = 0$.

Have students explain the difference between a line and a line segment. Write down a line and ask students what characteristics this line possesses. Repeat.

Distribute worksheets and assign problems from textbook Unit 3 "Relations and Functions" Distance-time and Velocity-time Graphs, p. 93; and Linear and Rational Relations p. 97.

(7) **Materials**: Graph paper, ruler, graphing calculator

(8) **Assessment**:
Draw some non-vertical lines and line segments and have students calculate the slopes.
Have the student calculate the slopes of lines making various angles with the x-axis, using a calculator.

III. **Lesson: Rate of Change, for Proportional Relationships**

(1) **Position in POS**: Relations and Functions Topic 3, continued

(2) **General Outcome**: Develop algebraic and graphical reasoning through the study of lines and linear relations.

(3) **Specific Outcomes**: Demonstrate an understanding of slope with respect to rate of change [PS, R, V]

(4) **Learning Objective**: Explain, using examples, slope as a rate of change.

(5) **Activity**:
Have students compare various lines with different slopes. What is slope saying about the way the dependent variable changes as the independent one does? Give real-world examples and have students explain what slope or rate of change is in each case:

- price/L of gasoline
- converting between units of measurement
- scale map
- speed
- thickness of a stack of paper in terms of number of sheets (approximately 0.05 mm/sheet)
- number of hockey cards per pack
- steepness of a road - The amount you go up is proportional to the amount you go across

Have students think about the word "per", both by itself and in the word "percent". What has this to do with proportionality?

Have students find the appropriate slope or rate-of-change given eg.

- \$30/6 t-shirts
- 5 map centimeters = 10 real km
- 40 km in half an hour
- 10 cm rainfall in a 14 day period
- Which is the better deal? 500 mL of Pepsi for \$0.89 or 2 L for \$2.79?

(6) **Materials**: Graph paper, ruler, graphing calculator

(7) **Assessment**:

- Many more problems of this nature.
- What happens when the constant of proportionality is negative?

IV. **Lesson: Thinking about linear relationships**

(1) **Position in POS**:
Relations and Functions Topic 1 [C, CN, R, T, V]

(2) **General Outcome**:
Develop algebraic and graphical reasoning through the study of lines and linear relations.

(3) **Specific Outcomes**:
Interpret and explain the relationships among data, graphs and situations.

(4) **Learning Objective**:
Gain insight into linear relationships between data.

(5) **Activity**:
This activity is from [1]. Divide the class into groups of 4-5 students and give each group a box of 20-30 toothpicks. Ask the groups to determine the number of toothpicks required to make the various shapes in Figure IV5 at the end of this document. Student should give an answer in terms of the dimension k of the figure.

- Connecting squares: k is the number of squares. Also find the length of the perimeter, in units of the length of a toothpick.
- Ladder: k is the number of squares before the two ends are removed.
- Truss: Find: 1) total number of triangles and 2) number of toothpicks for the truss, in terms of the number k of upright (Δ) triangles. Same questions in terms of the number of triangles pointing down.
- Square: 3 by 3 ($k = 3$) square shown. Students should find the number of 1 by 1 squares; number of toothpicks, and, in units of toothpicks, the length of the perimeter. Which of these is a linear function of k?

If the number of toothpicks is in a linear relation with k then if k goes up by one, the number of toothpicks should go up by the same amount m, regardless of the initial value of k.

Another Activity: Have the students think about the following problem:

You have a certain volume of a fluid such as water, alcohol or mercury and you want to take very sensitive measurements of the volume as it changes over time. Why might you want to do this? How would you set it up? What can you do to make the measurements more sensitive, ie. accurate? Assume that the changes in volume over time are very small. Hint: You have already seen a solution to this problem.

As an instructor I am hoping to hear about novel approaches to this problem; ie. not ones that I have already seen or thought about.

Problem: If the bulb of a thermometer has volume 0.1 mL, and the tube has diameter 1 mm, plot the height of the mercury in the column as a function of volume.

Problem: Have students come up with examples and non-examples of linear relationships.

(6) **Materials**: Toothpicks, graph paper

(7) **Assessment**: Have students come up with other shapes.

V. **Lesson: Thinking about the y-intercept in linear relationships**

(1) **Position in POS**:
Relations and Functions, Topic 1 [C, CN, R, T, V]

(2) **General Outcome**:
Develop algebraic and graphical reasoning through the study of lines and linear relations.

(3) **Specific Outcomes**:

- 1 Graph, with or without technology, a set of linear data
- Explain why data points should or should not be connected on the graph for a situation.
- Describe a possible situation for a given graph.
- Sketch a possible graph for a given situation.
- Relate a linear graph, a set of linearly-related ordered pairs or a table of linear values.

(4) **Learning Objective**: Gain insight into information about a line other than slope. Learn how lines can be used to model various situations more general than just proportions.

(5) **Activity**:
If Bob's house is 300 m down the street to the east from Kim's house, describe position relative to Bob's house in terms of position relative to Kim's house.

Putting it together: What if position relative to Bob's house is in meters and position relative to Kim's house is in kilometers?

(6) **Materials**:
ruler, graph paper, graphing calculator

(7) **Assessment**:

- Have students use graphing calculator to graph lines for various values of slope m and y-intercept b. Encourage students to think about the relationship between this graphical and numerical data.

VI. **Lesson: Run and Rise**

(1) **Position in POS**:
Relations and Functions Topic 3. [PS, R, V]

(2) **General Outcome**:
Develop algebraic and graphical reasoning through the study of lines and linear relations.

(3) **Specific Outcomes**:
Demonstrate an understanding of slope m with respect to run and rise.

(4) **Learning Objective**:
Determine the slope of a non-vertical line segment by measuring or calculating the rise and run.

(5) **Pre-Assessment**:
Give an exercise from the textbook which allows students to demonstrate their understanding of the correspondence between ordered pairs and points of the Cartesian plane.

(6) **Activity**:
Draw a non-vertical line on the board and 2 distinct points on that line. Have the students think about concepts like run, rise and slope.

Assign problems from textbook.

(7) **Materials**:
Graph paper, ruler, graphing calculator

(8) **Assessment**:
Class participation; problems from textbook, calculating slope from linear data in various forms.

VII. **Lesson: Lines that do not pass through the origin**

(1) **Position in POS**:
Relations and Functions Topic 3, continued [PS, R, V]

(2) **General Outcome**:
Develop algebraic and graphical reasoning through the study of lines and linear relations.

(3) **Specific Outcomes**:
Understand what is meant by the slope of a line, even if it does not pass through the origin.

(4) **Learning Objectives**:

- Calculate the slope of a given line.
- Draw a line, given its slope and a point on the line.
- Find another point on a line, given the slope and a point on the line.

(5) **Activities**:

- Give 5-6 examples for each of these types of objectives for students to work on.
- Ask students to come up with one example of each to give to a partner to solve.
- Discuss conversion between temperature units of Celsius, Fahrenheit and Kelvin. See Figure VII5 at the end of this document.
- If Remembrance Day, November 11 is a Sunday, what day of the week did October 7 fall on?

(6) **Materials**:
Graph paper, ruler, graphing calculator

(7) **Assessment**:
Class participation; problems (from textbook):

- Calculate the slope of a given line.
- Draw a line, given its slope and a point on the line.
- Find another point on a line, given the slope and a point on the line.

VIII. **Lesson: Parallel and Perpendicular Lines**

(1) **Position in POS**:
Relations and Functions Topic 3, continued [PS, R, V]

(2) **General Outcome**: Develop algebraic and graphical reasoning through the study of lines and linear relations.

(3) **Specific Outcomes**: Demonstrate an understanding of slope with respect to parallel and perpendicular lines.

(4) **Learning Objective**: Generalize and apply a rule for determining whether two lines are parallel or perpendicular.

Write the equation of a linear relation, given the coordinates of a point on the line and the equation of a parallel or perpendicular line, and explain the reasoning.

(5) **Activity**:
Ask students what happens to the slope when we replace a line by one that is parallel/perpendicular to it.

Show students the example http://frink.machighway.com/ dynamicm/hunt.pdf

(6) **Materials**: Graph paper, ruler, graphing calculator

(7) **Assessment**:
Class participation; problems from textbook, such as:

- equation of a line, given a point on the line and the equation of a parallel/perpendicular line.
- equation of a line, given a point on the line and two points on a parallel/perpendicular line.

IX. **Lesson: Describe and represent linear relations**

(1) **Position in POS:** Relations and Functions Topic 4. [C, CN, R, V]

(2) **General Outcome**:
Develop algebraic and graphical reasoning through the study of lines and linear relations.

(3) **Specific Outcomes**:
Describe and represent linear relations, using:

- words
- ordered pairs
- tables of values
- graphs
- equations.

(4) **Learning Objectives**:

- Express a linear relation in different forms, and compare the graphs.
- Rewrite a linear relation in either slopeintercept or general form.
- Generalize and explain strategies for graphing a linear relation in slopeintercept, general or slopepoint form.
- Graph, with and without technology, a linear relation given in slopeintercept, general or slopepoint form, and explain the strategy used to create the graph.
- Identify equivalent linear relations from a set of linear relations.
- Match a set of linear relations to their graphs.

(5) **Pre-Assessment**:
Give a few problems from the textbook which test understanding of previous lessons

(6) **Activity**:
Ask students what happens to the slope when we replace a line by one that is parallel/perpendicular to it.

Assign problems from textbook

(7) **Materials**:
Graph paper, ruler, graphing calculator

(8) **Assessment**:
Class participation; problems from textbook.

X. **Lesson: Characteristics of the graphs of linear relations**

(1) **Position in POS**:
Relations and Functions, Topic 5 [CN, PS, R, V]

(2) **General Outcome**:
Develop algebraic and graphical reasoning through the study of lines and linear relations.

(3) **Specific Outcomes**:
Determine the characteristics of the graphs of linear relations, including the:

- intercepts
- slope
- domain
- range.

to solve problems.

(4) **Learning Objective**:

- Determine the intercepts of the graph of a linear relation, and state the intercepts as values or ordered pairs.
- Determine the slope of the graph of a linear relation.
- Determine the domain and range of the graph of a linear relation.
- Sketch a linear relation that has one intercept, two intercepts or an infinite number of intercepts.
- Identify the graph that corresponds to a given slope and y-intercept.
- Identify the slope and y-intercept that correspond to a given graph.
- Solve a contextual problem that involves intercepts, slope, domain or range of a linear relation.

(5) **Activity**: Give problems of this nature for students to work on, with or without graphing calculator.

(6) **Materials**: Graph paper, ruler, graphing calculator

(7) **Assessment**:
Class participation; problems from textbook which test these Learning Objectives.

XI. **Lesson: Ways of Expressing Linear Relations**

(1) **Position in POS**:
Relations and Functions Topic 6 [CN, R, T, V]

(2) **General Outcome**:
Develop algebraic and graphical reasoning through the study of lines and linear relations.

(3) **Specific Outcomes**:
Relate linear relations expressed in:

- slope-intercept form ($y = mx + b$)
- general form ($Ax + By + C = 0$)
- slope-point form ($y - y_1 = m(x - x_1)$)

to their graphs.

(4) **Learning Objectives**:

- Express a linear relation in different forms, and compare the graphs.
- Rewrite a linear relation in either slopeintercept or general form.
- Generalize and explain strategies for graphing a linear relation in slopeintercept, general or slopepoint form.
- Graph, with and without technology, a linear relation given in slopeintercept, general or slopepoint form, and explain the strategy used to create the graph.
- Identify equivalent linear relations from a set of linear relations.
- Match a set of linear relations to their graphs.

(5) **Activity**:
Have the students answer the question: We have seen that rise/run does not depend on the two points. Given $y = mx + b$, does rise/run depend on b?

Have students pinpoint the relationship between m and rise/run.

(6) **Materials**: Graph paper, ruler, graphing calculator

(7) **Assessment**:
Class participation; problems from textbook.

XII. **Lesson: Finding the Equation of a Linear Relation**

(1) **Position in POS**:
Relations and Functions Topic 7 [CN, PS, R, V]

(2) **General Outcome**:
Develop algebraic and graphical reasoning through the study of lines and linear relations.

(3) **Specific Outcomes**:
Determine the equation of a linear relation, given

- a graph,
- a point and the slope
- two points
- a point and the equation of a parallel or perpendicular line

to solve problems. [CN, PS, R, V]

(4) **Learning Objective**:
Determine the slope and y-intercept of a given linear relation, and write the equation in the form $y = mx + b$, from data of each of the four types in "Specific Outcomes".

(5) **Activity**:
Draw a non-vertical line on the board and 2 distinct points on that line. Have the students think about concepts like run, rise and slope.

(6) **Materials**: Graph paper, ruler, graphing calculator

(7) **Assessment**:
Class participation; problems from textbook.

XIII. **Lesson: Function Notation and Review**

(1) **Position in POS**: Relations and Functions Topic 8 [CN, ME, V]

(2) **General Outcome**: Develop algebraic and graphical reasoning through the study of lines and linear relations.

(3) **Specific Outcomes**: Represent a linear function, using function notation.

(4) **Learning Objective**:

- Express the equation of a linear function relating one variable to another one, using function notation.
- Express an equation given in function notation as a linear function in two variables.
- Determine the related range value, given a domain value for a linear function
- Determine the related domain value, given a range value for a linear function
- Sketch the graph of a linear function expressed in function notation.
- Improve fluency with topics presented throughout the lesson plan.

(5) **Pre-Assessment:**: Give students some data and ask them to translate it into different forms, with or without graphing calculator.

(6) **Activity:** Have students work on problems such as:

- If $f(x) = 3x - 2$, determine $f(1)$.
- If $g(t) = 7 + t$, determine t so that $g(t) = 15$.

(7) **Materials:** Graph paper, ruler, graphing calculator

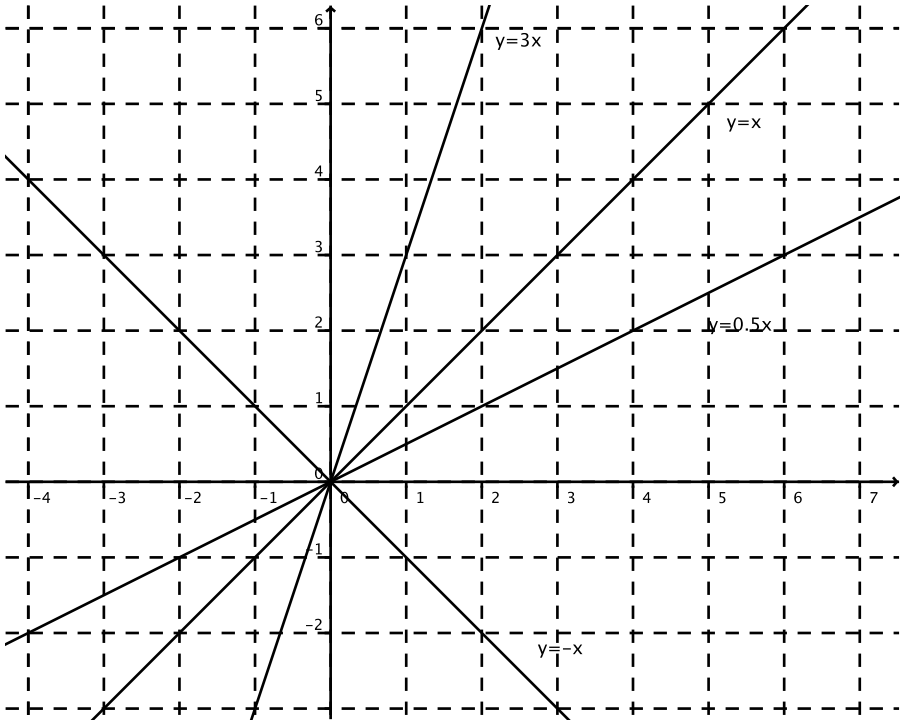
(8) **Assessment:** Class participation; more problems from textbook like the ones above.

References

[1] Bezuszka, Stanley J., and Margaret J. Kenney "The Three R's: Recursive Thinking, Recursion, and Recursive Formulas" in Algebra and Algebraic Thinking in School Mathematics: Seventieth Yearbook; Carole E. Greenes, Rheta Rubenstein, eds. National Council of Teachers of Mathematics (Reston, VA, 2008) pp. 81-98.

[2] "Mathematics 10" McGraw-Hill-Ryerson 2010.

[3] Alberta Math Program of Studies 2008 http://education.alberta.ca/media/823110/math10to12_ind.pdf


y=3x
y=x
y=0.5x
y=-x
6
5
4
3
2
1
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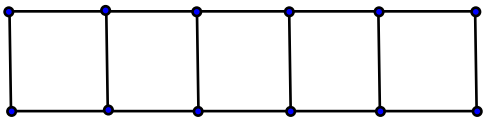
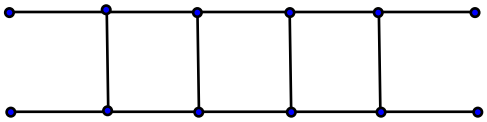
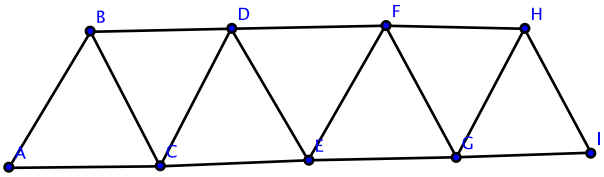


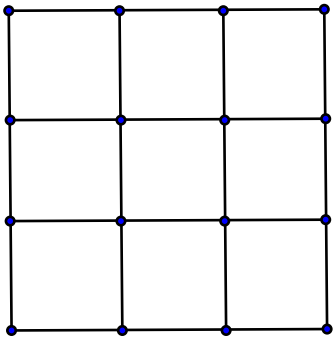
k touching squares
(shown for k=5)



ladder- k touching squares
with ends removed
(shown for k=5)



truss



square (k is dimension
of the square). (shown for
$k=3$)

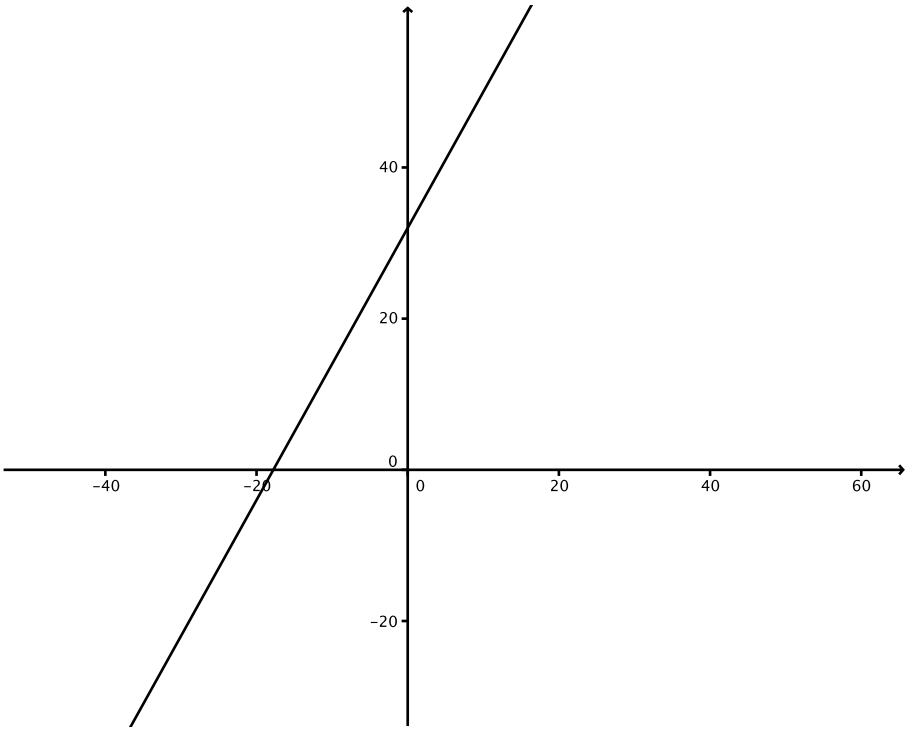


Figure 1: The x-axis is temperature in degrees Celsius; the y-axis in degrees Fahrenheit